Modern Trends in PDE's, Geometric Analysis and Mathematical Physics

University of Cergy–Pontoise

5–6 September 2011

Jean Bellissard. *Wannier transform for aperiodic solids: preliminary results*

The talk will present some preliminary results on a work in progress concerning the possible extension of the Bloch theory to aperiodic Schrödinger operators on $L^2(\mathbb{R}^d)$. The operator giving the potential energy in this specific case will be seen as a sum of short range atomic potentials with atoms located on a Delone set $\mathcal{L} \subset \mathbb{R}^d$, which is aperiodic, repetitive with finite local complexity. The main properties of such sets will be illustrated on the example of the Fibonacci sequence. But the Penrose, the octagonal tilings and the icosahedral phase of several quasicrystals are important other examples. The Anderson-Putnam complex and the Lagarias group will be defined. This will lead to a definition of the Wannier transform that is shown to be a unitary map onto a Hilbert space analog to the space of Bloch functions used in Solid State Physics. The Schrödinger operator will be shown to have a Bloch like representation and its domain will be characterized. Comments will be made about the potential applications of such method of analysis on the spectral properties of such an operator.

Camillo De Lellis. *Quantitative rigidity estimates.*

For many classical rigidity questions in differential geometry it is natural to ask to which extent they are stable. I will review several results in the literature and mention a few applications.

Jürg Fröhlich. *Friction and diffusion in models of particles interacting with a dispersive medium*

I review recent results (with De Roeck, Pizzo, Soffer and Gang Zhou) on quantum Brownian motion and quantum friction in some models of a tracer particle coupled to gapless modes of a quantum mechanical reservoir (e.g., black body radiation). An example of such results is that if the reservoir is in thermal equilibrium at positive temperature, and in dimension greater than or equal to 3 (or 4), a heavy tracer particle with an internal degree of freedom that can be excited by absorption of modes from the reservoir exhibits a diffusive motion at large times.

Christian Gérard. *Existence and non-existence of ground states in Quantum Field Theory models.*

Dans cet exposé nous ferons une revue du problème de l'existence d'un état fondamental pour différents modèles de physique quantique non-relativiste.

Claude Le Bris. *Some recent progress in deterministic and stochastic homogenization of elliptic PDE's*

The talk will overview, from a mathematical and numerical perspective, some challenging problems related to contemporary multiscale materials science. Recent developments of mathematical methods in the area of (both deterministic and stochastic) homogenization will be described. The talk covers a series of joint works with X. Blanc (CEA) and P. L. Lions (College de France), F. Legoll, A. Anatharaman, R. Costaouec, and F. Thomines (Ecole des Ponts).

Terry Lyons. *The expected signature of a stochastic process. Some new PDE's*

How can one describe a probability measure of paths? And how should one approximate to this measure so as to capture the effect of this randomly evolving system. Markovian measures were efficiently described by Stroock and Varadhan through the Martingale problem. But there are many measures on paths that are not Markovian and a new tool, the expected signature provides a systematic ways of describing such measures in terms of their effects.

We explain how to calculate this expected signature I the case of the measure on paths corresponding to a Brownian motion started at a point x in the open set and run until it leaves the same set. A completely new (at least to the speaker) PDE is needed to characterise this expected signature. (Joint work with Ni Hao.)

Robert McCann. *Geometric Variational Problems in Economics.*

The monopolist's problem of deciding what types of products to manufacture and how much to charge for each of them, knowing only statistical information about the preferences of an anonymous field of potential buyers, is one of the basic problems analyzed in economic theory. The solution to this problem when the space of products and of buyers can each be parameterized by a single variable (say quality X, and income Y) garnered Mirrlees (1971) and Spence (1974) their Nobel prizes in 1996 and 2001. The multidimensional version of this question is a largely open problem in geometric analysis (see Basov's book "Multidimensional Screening".) I plan to describe recent work with A. Figalli and Y.-H. Kim, identifying structural conditions on the value $b(X, Y)$ of product X to buyer Y which reduce this problem to a convex program in a Banach space—leading to uniqueness and stability results for its solution, confirming robustness of certain economic phenomena observed by Armstrong (1996) such as the desirability for the monopolist to raise prices enough to drive a positive fraction of buyers out of the market, and yielding conjectures about the robustness of other phenomena observed Rochet and Chone (1998), such as the

clumping together of products marketed into subsets of various dimension. The passage to several dimensions relies on ideas from differential geometry / general relativity, optimal transportation, and nonlinear PDE.

Wendelin Werner. *Random surfaces and random geometry.*

The Gaussian Free field is one of the most classical building blocks of quantum field theory, and it can be viewed as describing the large-scale limit of various simple models of random surfaces. I will describe recent works (many of which by Scott Sheffield, sometimes together with coauthors including myself) that show how geometric structures are embedded in the two-dimensional Gaussian Free Field.