Stochastic and PDE methods in mathematical physics
University of Paris-Diderot
15 – 17 September 2014

Titles and abstracts

Minicourses

M. Gubinelli : *Paracontrolled distributions and SPDEs*

Abstract: I will explain how ideas from the theory of non-linear waves, namely the paradifferential calculus of Bony, can be used to tackle problems in the theory of SPDEs and more generally for problems involving non-linear operation on distributions. The aim of the minicourse is to develop the ideas and framework necessary to understand and give a meaning to singular (random) PDEs. Applications will include: differential equations driven by rough paths, the parabolic Anderson model in 2 dimensions, the Kardar–Parisi–Zhang equation, the equation of stochastic quantisation in 3d. We will outline also links with the recent theory of regularity structures of M. Hairer.

C. Liverani : *The martingale approach after Varadhan and Dolgopyat*

Abstract: According to the vulgate a chaotic system behaves like a random one. If this were the case, then probabilistic methods should be applicable to chaotic deterministic systems. I will show how this is indeed possible, provided one can make the above mentioned (vague) similarity into quantitatively precise one. Such a possibility will be illustrated in an extremely simple example. Yet, the method (due to Dolgopyat, adapting to the field of dynamical systems an approach pioneered, mainly, by Varadhan) can be applied to a much wider class of problems.

Invited lectures

T. Alazard : *Zakharov's program in water-waves theory*

Abstract: In 1968, Zakharov introduced a program of research which intends to understand the dynamics of small amplitude water waves, and especially the transfer of energy among the waves. This program led to considerable progress in physics or oceanography, as well as in mathematics. I will review some of the main observations made by Zakharov, and report on some recent results about the Cauchy problem which are directly inspired by his program.

T. Bodineau : *The Brownian motion as the limit of a deterministic system of hard-spheres*

Abstract: We consider a tagged particle in a diluted gas of hard spheres. Starting from an hamiltonian dynamics of particles in the Boltzmann-Grad limit, we will show that the tagged particle follows a Brownian motion after an appropriate rescaling. We use the linear Boltzmann equation as an intermediate level of description for one tagged particle in a gas close to global equilibrium. Joint work with I. Gallagher et L. Saint-Raymond.

N. Burq : *Weak solutions of dispersive equations and Gibbs measures*

Abstract:

E. Faou : *Landau damping in Sobolev spaces for the Vlasov-HMF model*

Abstract: We consider the Vlasov-HMF (Hamiltonian Mean-Field) model. We consider solutions starting in a small Sobolev neighborhood of a spatially homogeneous state satisfying a linearized stability criterion (Penrose criterion). We prove that these solutions exhibit a scaterring behavior to a modified state, which implies a nonlinear Landau damping effect with polynomial rate of damping. We will explain the relations between this result and normal form methods, and show some numerical experiments making links between this result and the 2D Euler equation.

A. Komech : *On global attractors of Hamilton nonlinear PDEs*

Abstract: Click here for the abstract.

L. Koralov : *Averaging, homogenization, and large deviation methods for the study of randomly perturbed dynamical systems and PDEs*

Abstract: In this talk we'll discuss several asymptotic problems that can be formulated in terms of PDEs and solved using probabilistic methods. The first set of problems concerns the asymptotic behavior of solutions to quasi-linear parabolic equations with a small parameter at the second order term. Here we employ an extension of the large-deviation theory of Freidlin and Wentzell. Another set of problems concerns equations with a small diffusion term, where the first-order term corresponds to an incompressible flow, possibly with a complicated structure of flow lines. Here we use an extension of the averaging principle. Finally, we'll consider equations with a small diffusion term with periodic coefficients in a large domain. Depending on the relation between the parameters, either averaging or homogenization need to be applied in order to describe the behavior of solutions. We'll discuss the transition regime. Different parts of the talk are based on joint results with D. Dolgopyat, M. Freidlin, M. Hairer, Z. Pajor-Guylai, and L. Tcheuko.

V. Nersesyan : *Large deviations and Gallavotti–Cohen principle for randomly forced PDE's*

Abstract: We consider a class of dissipative PDE's driven by a random forcing. It is well known that, under some non-degeneracy condition for the random force, the Markov process associated with the problem admits a unique stationary measure. We prove a large deviations principle from the stationary measure for the occupation measures of the Markov process in question. In the case of a rough noise, under some additional hypotheses for the underlying deterministic system, we establish a Gallavotti–Cohen type fluctuation relation for an entropy production functional. This talk is based on joint works with V. Jaksic, C.-A. Pillet, and A. Shirikyan.

T. Oh : *Invariant Gibbs measures for the defocusing NLS on the real line*

Abstract: In 1994, Bourgain constructed invariant Gibbs measures for NLS on the circle. Then, in 2000, he considered the limit of these invariant statistics, by taking larger and lager periods, and constructed unique solutions for the defocusing (sub-)cubic NLS on the real line. His result,

however, focuses on the construction of solutions and does not discuss the limiting Gibbs measures on the real line or their invariance.

In this talk, we construct Gibbs measures for the defocusing NLS on the real line as a stationary diffusion process in x. Then, we show that these Gibbs measures are invariant for the defocusing (sub-)quintic NLS on the real line. We also discuss the limit Gibbs measures for the Dirichlet boundary value problem on the real line as well as the half line, allowing us to construct new rough solutions in these settings.

This is a joint work with Jeremy Quastel (University of Toronto) and Philippe Sosoe (Harvard University).

L. Thomann : *On the resonant Hermite–Schrödinger equation*

Abstract: We consider the cubic nonlinear Schrödinger equation in 3D with harmonic trapping in two directions. This model is used in the description of cigar-shaped Bose-Einstein condensates. Following a recent approach of Hani-Pausader-Tzvetkov-Visciglia we show that this equation scatters to some completely resonant system. In this talk, we will present some statistical properties of this limit system which may give some information on the original equation.

This is a joint work with Pierre Germain and Zaher Hani.