Non-equilibrium Statistical Mechanics of Stochastic Systems

University of Cergy-Pontoise, Laboratory AGM

3 - 4 March 2015

Organisers: V. Jakšić, A. Shirikyan

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Schedule

Tuesday, 3 March		
10:30 – 11:30	C.-A. Pillet	*Entropic fluctuations in harmonic networks*
12:00 – 13:00	L. Bruneau	*Conductances and absolutely continuous spectrum of 1D Schrodinger operators*
Lunch break		
14:30 – 15:30	N. Cuneo	*Non-equilibrium steady states for classical chains of rotors/oscillators* (lectures 1 and 2)
15:45 – 16:45	N. Cuneo	
Wednesday, 4 March		
10:30 – 11:30	N. Cuneo	*Non-equilibrium steady states for classical chains of rotors/oscillators* (lectures 3 and 4)
11:45 – 12:45	N. Cuneo	
Lunch break		
14:30 – 15:30	A. Dymov	*Nonequilibrium statistical mechanics of crystals in medium*
15:45 – 16:45	D. Martirosyan	*Large deviations for stationary measures of stochastic nonlinear wave equation with smooth white noise*

Titles and abstracts

Four-hour minicourse

N. Cuneo (Université de Genève): *Non-equilibrium steady states for classical chains of rotors/oscillators*

Abstract: Non-equilibrium steady states (NESS) of classical systems have been studied for some time. In this minicourse, I will present some results and remaining challenges for the case of classical chains of rotors/oscillators connected at both ends to stochastic heat baths at different temperatures. I will first briefly review some properties of the NESS from the literature, which are mostly obtained numerically, perturbatively, or via some effective theories. Then I will turn to rigorous results. Proving the mere existence of a NESS is a challenge, and has only been achieved in some special cases. I will present some existing probabilistic proofs, which rely on understanding limiting dynamics at high energy (which drastically depends on the model) in order to construct a Lyapunov function. I will also present some ongoing research, and discuss the difficulties of the models for which there is no proof yet.

One-hour talks

L. Bruneau (Université de Cergy-Pontoise): *Conductances and absolutely continuous spectrum of 1D Schrödinger operators*

Abstract: We characterize the absolutely continuous spectrum of the one-dimensional Schrödinger operators $h = -\Delta + v$ acting on $\ell^2(Z_+)$ in terms of the limiting behaviour of the Landauer-Büttiker and Thouless conductances of the associated finite samples. The finite samples are defined by restricting h to a finite interval in $[1, L]$ and the conductance refers to the charge current across the sample in the open quantum system obtained by attaching independent electronic reservoirs to the sample ends. We prove that the conductances associated to an energy interval I are non-vanishing in the limit $L \to \infty$ (physical characterization of the metallic regime) if and only if $sp_{ac}(h) \cap I \neq \emptyset$ (mathematical characterization of the metallic regime). This is a joint work with V. Jaksic, Y. Last and C.-A. Pillet.

A. Dymov (Université de Cergy-Pontoise): *Nonequilibrium statistical mechanics of crystals in medium*

Abstract: Investigation of the energy transport in crystals is one of the main problems in the nonequilibrium statistical mechanics. Since it turns out to be extremely difficult, usually one studies toy models, possessing additional ergodic properties. A common idea is to consider a Hamiltonian system of particles where each mode is a subject to stochastic perturbation. Clearly, it is important to study the case when the perturbation goes to zero.

In this talk I will discuss dynamics of an anharmonic system of weakly interacting oscillators, where each oscillator is weakly coupled with its own stochastic Langevin thermostat. The system can be interpreted as a crystal plugged in medium and weakly interacting with it. I will explain the method of resonant averaging and of effective equation, and using them I will prove that, under the limit when the couplings of oscillators with each other and with the thermostats go to zero with some precise scaling, behaviour of the system is governed by an effective equation which is a rather nice dissipative SDE. I will show that under the limit above, dynamics of the energy satisfies laws, which resemble the Fourier law and the Green-Kubo formula (but which are not the F. law and the G.-K. formula).

D. Martirosyan (Université de Cergy-Pontoise): *Large deviations for stationary measures of stochastic nonlinear wave equation with smooth white noise.*

Abstract: We study the nonlinear wave equation perturbed by a white noise whose amplitude tends to zero. We are interested in the asymptotic behavior of the family of stationary measures associated with the flow of that equation. We prove that this family satisfies the large deviations principle, which means that there is a function that describes precisely the logarithmic asymptotics of that family. The proof relies on a development of Freidlin-Wentzell's approach.

C.-A. Pillet (Université de Toulon et Aix Marseille Université): *Entropic fluctuations in harmonic networks*

Abstract: In this joint work with V. Jaksic and A. Shirikyan we study the large deviations of the entropy production functional of a network of harmonic oscillator. We find explicit conditions which ensure that the cumulant generating function of the entropy production rate is steep on a maximal interval. In the special case of harmonic chains, we show that this maximal interval coincides with the interval on which Rey-Bellet and Thomas have proved existence and finiteness of the generating function for a more general class of anharmonic chains.